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Computational Fluid Dynamics Based on the Method of Space-Time Conservation Element and Solution Element

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Summary

We have been making a fruitful progress under the support of this Grant. Within a period of a year, we have applied the space-time conservation element and solution element (S-T CE/SE) method to not only the flow problems described in our proposal, but also those not originally included. All of the available numerical schemes, including both the 1-D and 2-D a-μ scheme, a-ϵ scheme and Euler solver, have been validated using selected test problems. Numerical results are compared with exact solutions and/or numerical solutions obtained by some traditional methods to demonstrate the effectiveness and advantages of this method. Furthermore, some numerical schemes are modified to become new schemes, such as the ν-ϵ scheme and the ν-μ scheme, and various boundary conditions being consistent with the present scheme are derived and tested with appropriate model problems. The following is a brief description of three pieces of work that have been completed so far.

1. Application to Shock-Tube Problems

The Euler solver is one of the numerical schemes constructed based on the S-T CE/SE method for solving the one-dimensional unsteady Euler equations. It was used to obtain accurate flow solutions for an infinitely long shock tube.

In this work, the Euler solver is extended and applied to more complex flow problems involving shock tubes of finite or infinite length. Seven test problems are used to examine the ability of the Euler solver, which result in high resolutions of shock waves, contact discontinuities and expansion waves. For the shock tube with infinite length, there exist different flow phenomena with varied initial conditions, such as strong shock waves moving in opposite directions, slowly moving contact surface and strong expansion fans. For shock tubes with finite length, the flow field may contain more complicated phenomena such as the reflection of shock waves from a rigid wall, merging of two shock waves and intersection between a shock wave and a contact surface. The simulation of such flow fields thus becomes more difficult. Numerical results, when compared with the exact and/or numerical solutions obtained by upwind schemes, show that the present Euler solver can generate highly accurate solutions in regions of steep gradients without using any ad hoc techniques. The accuracy, simplicity and generality of this method are verified here. This work has been reported in "Application of the Space-Time Conservation Element and Solution Element to Shock-Tube Problem" by X.Y. Wang, C.Y. Chow and S.C. Chang, NASA TM 106806, December 1994.

2. Application to One-Dimensional Advection-Diffusion Problems

The existing a-μ scheme is one of the basic numerical schemes that have been constructed based on the S-T CE/SE method for solving the unsteady linear advection-diffusion equation.

In this work, the 1-D steady advection-diffusion equation is solved by use of the a-μ scheme to demonstrate that it can also be utilized to generate highly accurate steady-state solutions. In addition, the 1-D unsteady diffusion equation is solved using the μ scheme, which is a special form of the a-μ scheme by letting $a = 0$; with such a change the equation type changes from hyperbolic to parabolic. Two test problems are solved here to investigate the new properties of the μ scheme which are different from those of the a-μ scheme. Furthermore, the a-μ scheme is modified to become, respectively, the ν-μ scheme for viscous Burgers equation and the ν-ϵ scheme for inviscid Burgers equation. Two test problems are used to demonstrate that the a-μ scheme is still far superior to several traditional finite-difference methods for solving nonlinear problems. Numerical results are compared with several traditional methods, such as the Galerkin finite element method, Dufort-Frankel explicit method and Crank-Nicolson implicit finite-difference method.

The comparison shows that numerical solutions obtained by the present method are generally more accurate than those based on many traditional finite-difference methods but with the same or less restrictive stability criteria. For further investigations, a more efficient implicit scheme based on the S-T CE/SE method for solving the unsteady diffusion equation will be developed.

3. Application to Two-Dimensional Advection-Diffusion Problems

The two-dimensional versions of the a-μ scheme and a-ϵ scheme, which have been constructed based on the same set of design principles, are extended and examined using some test problems in this part.

The 2-D unsteady advection-diffusion equation is solved by use of the a-μ scheme. The influence of diffusivity on the accuracy of the numerical solution is investigated by comparing with the exact solution. On the other hand, the 2-D pure advection equation is solved by the a-ϵ scheme, using different values of ϵ to study its effect on numerical diffusion. Large ϵ will lead to too much numerical diffusion. Then the 2-D pure diffusion equation is solved using the μ scheme, which is a special form of the a-μ scheme by letting $a_x = a_y = 0$. The numerical results so computed are compared with those obtained by Crank-Nicolson implicit finite-difference and finite-element methods. Furthermore, the a-ϵ scheme is modified to become the ν-ϵ scheme for inviscid Burgers equation. Two test problems are used to demonstrate that the ν-ϵ scheme is still far more superior to several traditional finite-difference methods for solving nonlinear problems in a much simpler manner.

Numerical solutions of typical test problems indicate again that the CE/SE method is still more superior to many finite-difference and finite-element methods in two-dimensional configurations. The accuracy, simplicity and generality of this method are verified again. A more efficient implicit scheme based on this method also needs to be developed in the 2-D case.